Accelerating dark-matter axion searches with quantum measurement technology

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arXiv:1607.02529
Take-home message:

The effort to build quantum computers has led to remarkable progress in superconducting qubits and Josephson parametric amplifiers.

We can use these new microwave quantum optics (‘circuit QED’) technologies to improve axion detection at high frequencies.

Proposal: Zheng et al., arXiv:1607.02529 (being revised)

1. Direct photon number counting
2. Two-mode Squeezing
Conversion of axions to microwave photons

\[ H_{\text{conversion}} = g_{d\gamma\gamma} (d + d^\dagger) \vec{E}_{\text{microwave}} \cdot \vec{B}_{\text{dc}} \]

\[ 1 \lesssim m_a \lesssim 100 \mu eV \Rightarrow 250 \text{ MHz} \lesssim \nu \lesssim 25 \text{ GHz} \]

\[ (21 \text{ GHz} \approx 1K) \]

(axion virialization) \[ Q_a \sim 10^6 \]

(normal metal cavity) \[ Q_c \sim 10^4 - 10^5 \]

Production rate of microwave photons is low;
Mean cavity occupancy: \[ \bar{n} \sim 10^{-3} - 10^{-5} \]
HAYSTAC Experiment uses a nearly quantum-limited Josephson Parametric Amplifier (JPA).

Ben Brubaker et al., *PRL* 118, 061302 (2017) 5.7 GHz

Quantum amplifiers (and attenuators) operate under constraints due to unitarity.

Even perfect amplifiers generically add noise which overwhelms the ultraweak axion signal.

\[
\bar{n}_{\text{axion}} \ll \frac{1}{2} + \bar{n}_{\text{amp}} + \bar{n}_T
\]

Zero-point energy
Amplifier added noise
Thermal noise

\[
\bar{n}_{\text{amp}} \geq \frac{1}{2}
\]

SQL

HAYSTAC: \( \bar{n}_{\text{amp}} \sim 1.3 - 2.5 \)
Circuit QED:
- Artificial Josephson junction ‘atoms’ (SC qubits)
- Coupled to individual microwave photons
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- We routinely build ‘photomultipliers’ to count individual microwave photons.
- Measuring $\hat{n} = a^\dagger a$ avoids the vacuum noise.
Circuit QED:
- Artificial Josephson junction ‘atoms’ (SC qubits)
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- We routinely build ‘photomultipliers’ to count individual microwave photons.
- Measuring $\hat{n} = a^\dagger a$ avoids the vacuum noise.

- Unlike a photomultiplier, we can do QND measurements that can be repeated 10-100 times, dramatically enhancing the quantum efficiency and lowering the dark count.
Transmon Qubit

Pseudo-spin-1/2

$|0\rangle = \downarrow$
$|1\rangle = \uparrow$

$H = \frac{\omega_0}{2} \sigma^z$

$\omega_0 \sim 5 - 10 \text{ GHz}$

10^{11}$ mobile electrons

Superconductivity gaps out single-particle excitations

Quantized energy level spectrum is \textit{simpler} than hydrogen

Quality factor $Q = \omega T_1$ \textit{exceeds} that of hydrogen
Transmon Qubit in 3D Cavity

Huge dipole moment: strong coupling

$$g = \frac{\vec{d} \cdot \vec{E}_{\text{rms}}}{\hbar}$$

$$|\vec{d}| = 2e \times 1 \text{ mm} \approx 10^7 \text{ Debye}!!$$

$$V_{\text{dipole}} = g \sigma^x (a + a^\dagger)$$

$$g \sim 100 \text{ MHz}$$
Quantum optics at the single photon level

Large dipole coupling of transmon qubit to cavity enables:

‘Strong-Dispersive’ Hamiltonian: qubit detuned from cavity
-qubit can only virtually absorb/emit photons

(DOUBLY QND)

\[ H = \omega_r a^\dagger a + \frac{\omega_q}{2} \sigma^z + \chi \sigma^z a^\dagger a + H_{\text{damping}} \]

resonator    qubit    Dispersive coupling

Together with drives on the qubit and the cavity, strong-dispersive Hamiltonian gives universal controllability.
Deterministic Photon Cat Production

Example of universal controllability:

Strong-Dispersive Hamiltonian

\[ H = \omega_r a^\dagger a + \frac{\omega_q}{2} \sigma^z + \chi \sigma^z a^\dagger a + H_{\text{damping}} \]

resonator  qubit  dispersive coupling

cavity frequency = \omega_r + \chi \sigma^z

Very easy to readout state of qubit with 99.5% fidelity in 300-400ns by measuring the cavity frequency. (QND)
Strong-Dispersive Hamiltonian

\[ H = \omega_r a^\dagger a + \frac{\omega_q}{2} \sigma^z + \chi \sigma^z a^\dagger a + H_{\text{damping}} \]

resonator  qubit  dispersive coupling

\[ H = \omega_r a^\dagger a + \frac{\omega_q + 2\chi a^\dagger a}{2} \sigma^z + H_{\text{damping}} \]

Quantized light shift of qubit frequency

Easy to measure the photon number in the cavity by measuring the qubit transition frequency.
Usual (absorption) spectroscopy

Quantum jump spectroscopy of qubit gives QND measurement of storage cavity photon number

Axion signal line

high Q superconducting storage cavity

$Q \sim 10^5$

low Q readout cavity

Transmon qubit dispersively coupled to both cavities

$\omega_{\text{spectroscopy}}$

$\omega_{\text{readout}}$

$(\pi \text{ pulse})$
Quantum Jump Spectroscopy of the Qubit to detect photons in the storage cavity

\[ H = \omega_s a^\dagger a + \omega_q |e\rangle\langle e| - \chi_{sa} a^\dagger a |e\rangle\langle e| - K_{ss} (a^\dagger a)^2 \]

Ancilla (transmon) readout fidelity ~ 99.5% in 400 ns. QND for both qubit and storage cavity.

Quantum-limited amplifier (Devoret lab)
Microwaves are particles!

- quantized light shift of qubit frequency
  (coherent microwave state)

\[ \frac{\omega_q + 2\chi a^\dagger a}{2\sigma^z} \]

New low-noise way to do axion dark matter detection using quantum jump spectroscopy of qubit?
Thermal photon number

For 5 GHz photons in equilibrium at T=20mK, \( n_T \sim 3 \times 10^{-6} \)

From measurement of the quantized light shift we can bound the number of thermal photons \( n_T < 2 \times 10^{-3} \)


\[ \bar{n}_{\text{axion}} \sim 10^{-3} - 10^{-5} \]

Qubit readout fidelity 99.5% implies dark count \( \bar{n}_{\text{dark}} \sim 5 \times 10^{-3} \)
We can take advantage of the fact that the photon in the storage cavity is not absorbed during the QND measurement.

Ancilla (transmon) readout fidelity ~ 99.5% in 400 ns

\[ H = \omega_r a^\dagger a + \frac{\omega_q}{2} \sigma^z + \chi \sigma^z a^\dagger a + H_{\text{damping}} \]

(resonator) qubit Dispersive coupling

Quantum-limited amplifier (Devoret lab)
Thermal photon number

For 5 GHz photons in equilibrium at $T=20\text{mK}$, 

\[ n_T \sim 3 \times 10^{-6} \]

From measurement of the quantized light shift we can bound the number of thermal photons 

\[ n_T < 2 \times 10^{-3} \]


\[ \overline{n}_{\text{axion}} \sim 10^{-3} - 10^{-5} \]

Qubit readout fidelity $F=99.5\%$ implies dark count 

\[ \overline{n}_{\text{dark}} = 1 - F \sim 5 \times 10^{-3} \]

Measurement can be repeated $\sim5$-10 times in cavity lifetime.

**QND Readout**

Veto: require $m$ consecutive detections (not Bayes optimal)

False negatives: 

\[ F^m \sim (0.995)^m \]

False positives: 

\[ (1 - F)^m \sim (\overline{n}_{\text{dark}})^m \]

\[ \overline{n}_{\text{dark}} \sim (\overline{n}_{\text{dark}})^m \]
Conclusion: Scan rate speed-up relative to coherent detection

\[ R \sim \frac{\Delta \nu_{\text{axion}}}{\Delta \nu_{\text{cavity}}} \frac{1}{\frac{1}{2} \overline{n}_{\text{amp}} + \overline{n}_{T} + \overline{n}_{\text{axion}}} \sim 10^2 \]

Analysis presented here is a highly simplified tutorial.

Full details of optimal acausal Bayesian filter are presented in:

Zheng et al., arXiv:1607.02529 (currently being revised).
Magic of two-mode squeezing: vacuum noise is perfectly cancelled while axion signal is amplified by $U^\dagger$.

$S=10$ dB of squeezing has been achieved with current technology (KW Lehnert).

Risk: small transmission losses through the device will limit scan speed-up to approximately 4x unless they can be significantly reduced.
Circuit QED has strong-coupling microwave quantum optics has capabilities undreamt of in conventional optics.

The first electronic quantum processor (2009)

[Grover search algorithm, Deutsch Josza algorithm]

DiCarlo et al., Nature 460, 240 (2009)

Progress in quantum computation has spin-offs for table-top physics with skyscraper reach!

Many thanks to my collaborator Konrad Lehnert.
Linear Quantum Amplification and Two-Mode Squeezing
Quadrature Amplitudes of Quantum Signals are Incompatible Observables!

Voltage quadrature amplitudes are canonically conjugate:

\[ \hat{X}(t) = \hat{X}(0) \cos(\omega t) + \hat{Y}(0) \sin(\omega t) \]

Dimensionless amplitudes:

\[ \hat{X} = \frac{a + a^\dagger}{2}, \quad \hat{Y} = \frac{a - a^\dagger}{2i} \]

\[ [\hat{X}, \hat{Y}] = \frac{i}{2} \]

Ground (vacuum) state is minimum-uncertainty wave packet.

\[ \langle 0 \mid X^2 + Y^2 \mid 0 \rangle = \frac{1}{2} \]
Linear Phase-Preserving Amplification

\[ \hat{X}_{\text{out}} = g\hat{X}_{\text{in}} \]

\[ \hat{Y}_{\text{out}} = g\hat{Y}_{\text{in}} \]

\[
\begin{bmatrix}
\hat{X}_{\text{out}}, \hat{Y}_{\text{out}}
\end{bmatrix} = g^2 \begin{bmatrix}
\hat{X}_{\text{in}}, \hat{Y}_{\text{in}}
\end{bmatrix}
\neq \begin{bmatrix}
\hat{X}_{\text{in}}, \hat{Y}_{\text{in}}
\end{bmatrix}
\]

IMPOSSIBLE!! NOT CANONICAL
Linear Phase-Preserving Amplification

No amplifier ($g>1$) or attenuator ($g<1$) can ever do this:

\[ \hat{X}_{\text{out}} = g\hat{X}_{\text{in}} \]
\[ \hat{Y}_{\text{out}} = g\hat{Y}_{\text{in}} \]
We understand beam splitters as quantum attenuators.

\[
\begin{align*}
\begin{pmatrix}
a_{\text{out}} \\
b_{\text{out}}
\end{pmatrix}
&= \begin{pmatrix}
t & r \\
-r & t
\end{pmatrix}
\begin{pmatrix}
a_{\text{in}} \\
b_{\text{in}}
\end{pmatrix}
\begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
a_{\text{in}} \\
b_{\text{in}}
\end{pmatrix}
\end{align*}
\]

Conservation of probability = unitarity

\[ t^2 + r^2 = 1 \]

Unitarity requires the presence of a second (vacuum) port which adds (vacuum) noise.

\[
\begin{align*}
\left[ a_{\text{out}}, a_{\text{out}}^\dagger \right] &= \cos^2 \theta \left[ a_{\text{in}}, a_{\text{in}}^\dagger \right] + \sin^2 \theta \left[ b_{\text{in}}, b_{\text{in}}^\dagger \right] = 1
\end{align*}
\]
Quantum attenuators are **SU(2)** beam splitters

Signal port \( a_{in} \) \( a_{out} \) 

Vacuum port \( b_{in} \) \( b_{out} \)

Attenuated signal

\[
\begin{pmatrix}
    a_{out} \\
    b_{out}
\end{pmatrix}
=
\begin{pmatrix}
    t & r \\
    -r & t
\end{pmatrix}
\begin{pmatrix}
    a_{in} \\
    b_{in}
\end{pmatrix}
=
\begin{pmatrix}
    \cos \theta & \sin \theta \\
    -\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
    a_{in} \\
    b_{in}
\end{pmatrix}
\]

\[
\begin{pmatrix}
    a_{out} \\
    b_{out}
\end{pmatrix}
=
\begin{pmatrix}
    \cos^2 \theta a_{in} + \sin^2 \theta b_{in} \\
    \sin \theta \cos \theta a_{in} - \sin \theta \cos \theta b_{in}
\end{pmatrix}
\]

Total vacuum noise is unaffected by the beam splitter.

\[
\langle 0_{in} \mid X^2_{out} + Y^2_{out} \mid 0_{in} \rangle = \frac{1}{2}
\]
Quantum amplifiers are **SU(1,1) ‘beam splitters’**

\[
g^2 = G = \text{power gain}
\]

\[
\begin{pmatrix}
a_{\text{out}} \\
\bar{b}_{\text{out}}
\end{pmatrix} = \begin{pmatrix}
g & \frac{\sqrt{g^2 - 1}}{g} \\
\sqrt{g^2 - 1} & h
\end{pmatrix} \begin{pmatrix}
a_{\text{in}} \\
\bar{b}_{\text{in}}
\end{pmatrix} = \begin{pmatrix}
\cosh \theta & \sinh \theta \\
-\sinh \theta & \cosh \theta
\end{pmatrix} \begin{pmatrix}
a_{\text{in}} \\
\bar{b}_{\text{in}}
\end{pmatrix}
\]

\[
[a_{\text{out}}, a_{\text{out}}^\dagger] = \cosh^2 \theta [a_{\text{in}}, a_{\text{in}}^\dagger] + \sinh^2 \theta [\bar{b}_{\text{in}}, \bar{b}_{\text{in}}^\dagger] = \cosh^2 \theta - \sinh^2 \theta = 1
\]

Phase conjugation
Quantum amplifiers are **SU(1,1) ‘beam splitters’**

\[
g^2 = G = \text{power gain}
\]

\[
\begin{pmatrix}
a_{out} \\
\hat{b}_{out}^\dagger
\end{pmatrix} = \begin{pmatrix}
g & \sqrt{g^2 - 1} \\
\sqrt{g^2 - 1} & g
\end{pmatrix} \begin{pmatrix}
a_{in} \\
\hat{b}_{in}^\dagger
\end{pmatrix} = \begin{pmatrix}
\cosh \theta & \sinh \theta \\
-\sinh \theta & \cosh \theta
\end{pmatrix} \begin{pmatrix}
a_{in} \\
\hat{b}_{in}^\dagger
\end{pmatrix}
\]

Phase conjugation

\[
a_{out} = \sqrt{G} a_{in} + \sqrt{G - 1} \hat{b}_{in}^\dagger
\]

Output contains amplified vacuum noise from both ports. Noise is doubled (for G>>1).

½ quantum input noise
½ quantum minimum noise because of amplifying incompatible quadratures equally.
Dicke Radiometer SNR

\[ \alpha_{la} = \frac{1}{2} \frac{\eta \bar{n}_{\text{axion}}}{n_{\text{amp}} + \frac{1}{2} + \bar{n}_T} \sqrt{(\Delta \nu_c \tau) \left( \frac{\Delta \nu_c}{\Delta \nu_{\text{axion}}} \right)} \]

\[ \approx \frac{\bar{n}_{\text{axion}} \Delta \nu_c}{2} \sqrt{\frac{\tau}{\Delta \nu_{\text{axion}}}} \quad \text{(ideal limit)} \]

\[ Q_a \sim 10^6 \]
\[ Q_c \sim 10^4 - 10^5 \]
Quantum amplifiers are **SU(1,1) ‘beam splitters’**

![Diagram of quantum amplifiers]

**Equation:**

\[
g^2 = G = \text{power gain}
\]

\[
a_{\text{out}} = \sqrt{G} a_{\text{in}} + \sqrt{G-1} b_{\text{in}}
\]

\[
\bar{n} = \langle a_{\text{out}}^{\dagger} a_{\text{out}} \rangle = G \langle a_{\text{in}}^{\dagger} a_{\text{in}} \rangle + (G-1) \langle b_{\text{in}}^{\dagger} b_{\text{in}} \rangle = G - 1
\]

\[
\bar{n} = \langle b_{\text{out}}^{\dagger} b_{\text{out}} \rangle = G \langle b_{\text{in}}^{\dagger} b_{\text{in}} \rangle + (G-1) \langle a_{\text{in}}^{\dagger} a_{\text{in}} \rangle = G - 1
\]

Amplified vacuum noise yields a thermal state (**Hawking radiation**) in each port.
Amplified vacuum noise yields a thermal state (Hawking radiation) in each port.

Thermal noise in each output port has large entropy.

Yet the outputs have subtle quantum correlations due to two-mode squeezing:

\[ \langle a_{\text{out}} b_{\text{out}} \rangle = \sqrt{G(G-1)} \]
An ideal amplifier performs a unitary transformation from the input state to the output state.

This preserves the entropy of the vacuum input state (which is zero).

Quantum entanglement of the two output beams means that each is a thermal state with positive entropy yet the entropy of the universe is still zero!

Negative entropy of quantum entanglement.
Ideal amplification is a **unitary** process, so can be reversed.

\[
U(\theta) \leftrightarrow \begin{pmatrix} \cosh \theta & \sinh \theta \\ -\sinh \theta & \cosh \theta \end{pmatrix}
\]

\[
U^\dagger(\theta) = U(-\theta)
\]

\[
UU^\dagger = 1
\]

vacuum in -- vacuum out!
Near perfect quantum correlations between signal and idler lead to destructive interference.

Proof that the amplifier adds almost no entropy to the universe.
Magic of two-mode squeezing: vacuum noise is perfectly cancelled while axion signal is amplified by $U^\dagger$.

S=10 dB of squeezing has been achieved with current technology (KW Lehnert).

Risk: small transmission losses through the device will limit scan speed-up to approximately 4x unless they can be significantly reduced.
Extra Slides
3-wave mixing non-linearity

\[ \hat{H}_1 = i\hbar \chi \left( \hat{c} \hat{a}^\dagger \hat{b}^\dagger - \hat{c}^\dagger \hat{a} \hat{b} \right) \]

Energy conservation

\[ \omega_{\text{pump}} = \omega_{\text{signal}} + \omega_{\text{idler}} \]
HAYSTAC First Results

The graph shows the comparison of different theoretical models with experimental data, specifically focusing on the ratio of the axial vector coupling $g^\gamma_e / g^\text{KSVZ}_e$ as a function of the mass $m_a$ in units of microelectronvolts ($\mu$eV). The models include ADMX, UF, RBF, KSVZ, and DFSZ, while the graph also highlights the results from HAYSTAC's 'This work' experiment.

The x-axis represents the mass $m_a$ in $\mu$eV, ranging from 0 to 25, and the y-axis represents the ratio $g^\gamma_e / g^\text{KSVZ}_e$, ranging from $10^{-2}$ to $10^4$. The graph indicates a close agreement between the experimental data and the KSVZ model, while the DFSZ model shows a significant deviation from the experimental results.