

# Accelerating dark-matter axion searches with quantum measurement technology

Steven M. Girvin  
Yale Quantum Institute

Huaxiu Zheng  
Matti Silveri  
RT Brierley  
SM Girvin  
KW Lehnert,

[arXiv:1607.02529](https://arxiv.org/abs/1607.02529)



[QuantumInstitute.yale.edu](http://QuantumInstitute.yale.edu)

## Take-home message:

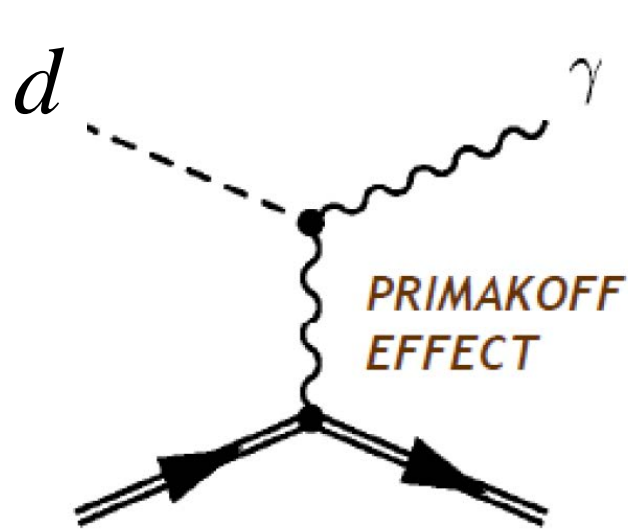
The effort to build quantum computers has led to remarkable progress in superconducting qubits and Josephson parametric amplifiers.

We can use these new microwave quantum optics ('**circuit QED**') technologies to improve axion detection at high frequencies.

Proposal: Zheng et al., [arXiv:1607.02529](https://arxiv.org/abs/1607.02529) (being revised)

1. Direct photon number counting
2. Two-mode Squeezing

# Conversion of axions to microwave photons



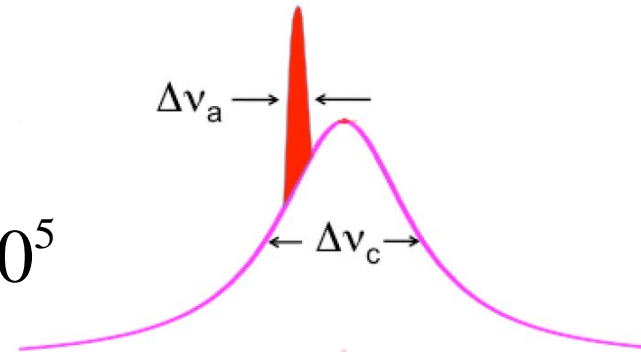
$$H_{\text{conversion}} = g_{d\gamma\gamma} (d + d^\dagger) \vec{E}_{\text{microwave}} \cdot \vec{B}_{\text{dc}}$$

$$1 \lesssim m_a \lesssim 100 \mu\text{eV} \Rightarrow 250 \text{ MHz} \lesssim \nu \lesssim 25 \text{ GHz}$$

(21 GHz  $\approx$  1K)

(axion virialization)  $Q_a \sim 10^6$

(normal metal cavity)  $Q_c \sim 10^4 - 10^5$



Production rate of microwave photons is low;

Mean cavity occupancy:  $\bar{n} \sim 10^{-3} - 10^{-5}$



# HAYSTAC Experiment



# Uses a nearly quantum-limited Josephson Parametric Amplifier (JPA)



Quantum amplifiers (and attenuators) operate under constraints due to unitarity.

Even perfect amplifiers generically add noise which overwhelms the ultraweak axion signal.

$$\bar{n}_{\text{axion}} \ll \frac{1}{2} + \bar{n}_{\text{amp}} + \bar{n}_{\text{T}}$$

Zero-point energy  
Amplifier added noise  
Thermal noise

$$\bar{n}_{\text{amp}} \geq \frac{1}{2} \quad \text{SQL}$$

HAYSTAC:  $\bar{n}_{\text{amp}} \sim 1.3 - 2.5$

Ben Brubaker et al., *PRL* **118**, 061302 (2017)  
5.7 GHz

## Circuit QED:

- Artificial Josephson junction 'atoms' (SC qubits)
- Coupled to individual microwave photons

## Circuit QED:

- Artificial Josephson junction 'atoms' (SC qubits)
- Coupled to individual microwave photons
  
- We routinely build 'photomultipliers' to count individual microwave photons.
- Measuring  $\hat{n} = a^\dagger a$  avoids the vacuum noise.

## Circuit QED:

- Artificial Josephson junction 'atoms' (SC qubits)
- Coupled to individual microwave photons
  
- We routinely build 'photomultipliers' to count individual microwave photons.
- Measuring  $\hat{n} = a^\dagger a$  avoids the vacuum noise.
  
- Unlike a photomultiplier, we can do QND measurements that can be repeated 10-100 times, dramatically enhancing the quantum efficiency and lowering the dark count.

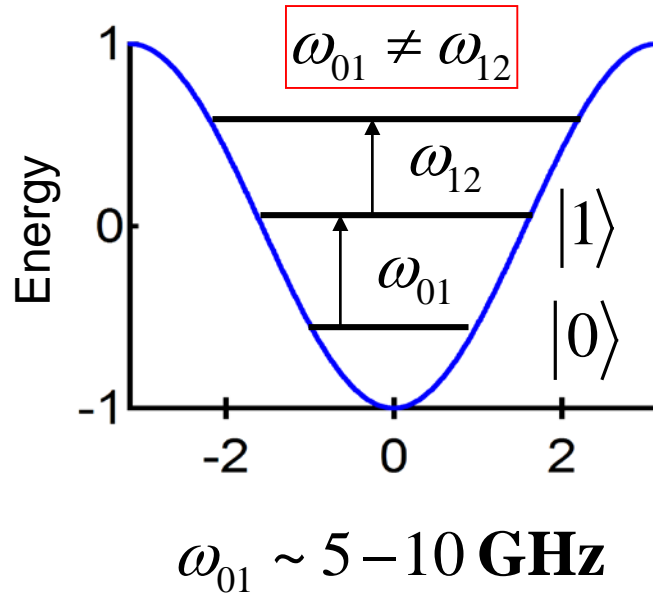
# Transmon Qubit

Pseudo-spin-1/2

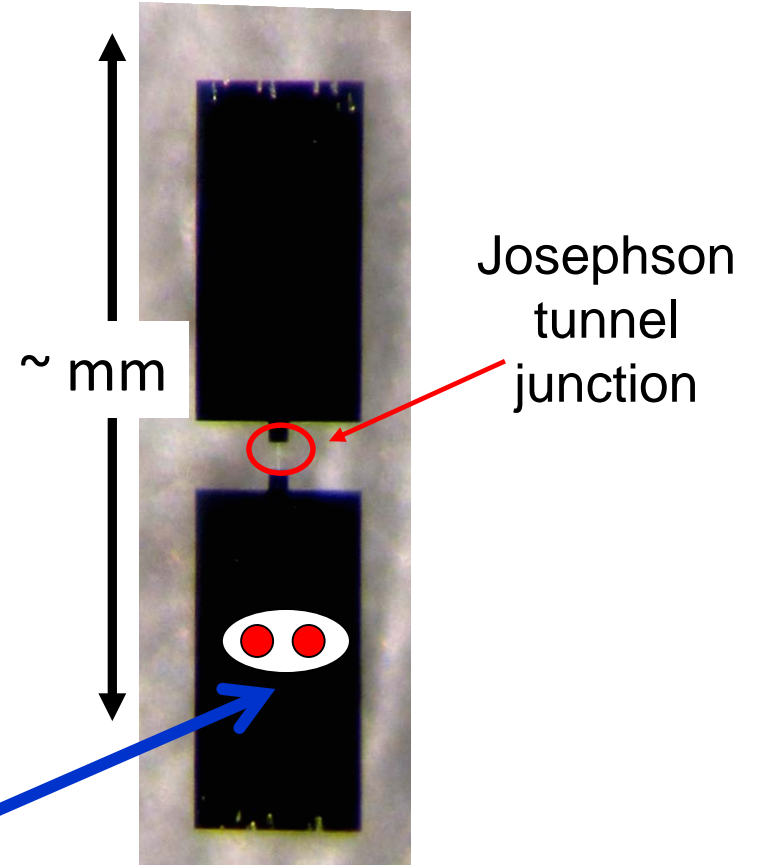
$$|0\rangle = |\downarrow\rangle$$

$$|1\rangle = |\uparrow\rangle$$

$$H = \frac{\omega_{01}}{2} \sigma^z$$



$10^{11}$  mobile electrons



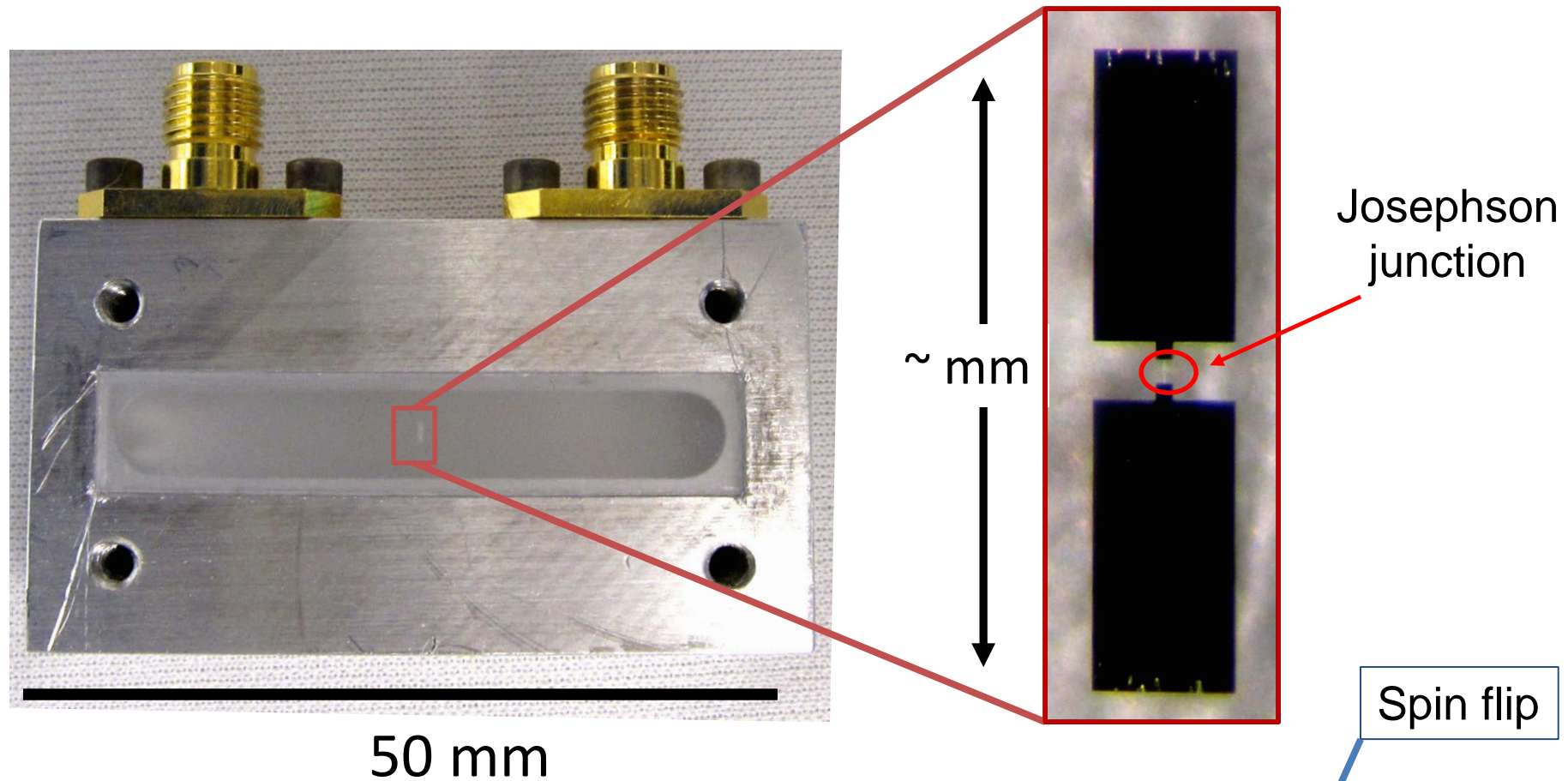
Superconductivity gaps out single-particle excitations

Quantized energy level spectrum is simpler than hydrogen

Quality factor  $Q = \omega T_1$  exceeds that of hydrogen



# Transmon Qubit in 3D Cavity



$$g = \frac{\vec{d} \cdot \vec{E}_{rms}}{h}$$

$$|\vec{d}| = 2e \times 1 \text{ mm} \approx 10^7 \text{ Debye!!}$$

$$V_{\text{dipole}} = g \sigma^x (a + a^\dagger)$$

Huge dipole moment: strong coupling

$$g \sim 100 \text{ MHz}$$

# Quantum optics at the single photon level

Large dipole coupling of transmon qubit to cavity enables:

‘Strong-Dispersive’ Hamiltonian: qubit detuned from cavity  
-qubit can only virtually absorb/emit photons

(DOUBLY QND)

$$H = \omega_r a^\dagger a + \frac{\omega_q}{2} \sigma^z + \chi \sigma^z a^\dagger a + H_{\text{damping}}$$

$$\omega_r \neq \omega_q$$

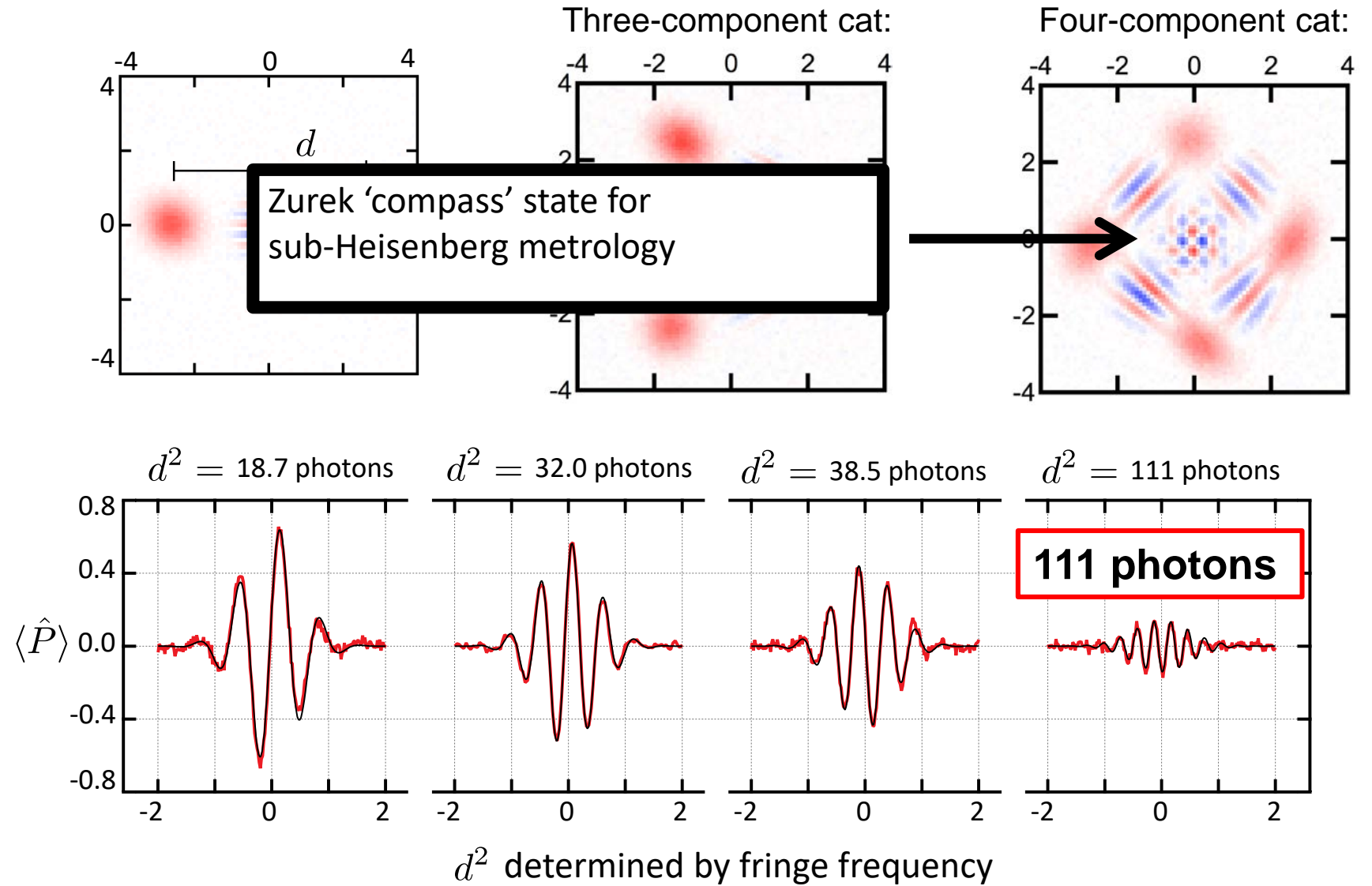
resonator      qubit      Dispersive coupling

Together with drives on the qubit and the cavity, strong-dispersive Hamiltonian gives universal controllability.

# Deterministic Photon Cat Production

Example of universal controllability:

Vlastakis, Kirchmair, et al.,  
*Science* (2013)



# Strong-Dispersive Hamiltonian

$$H = \omega_r a^\dagger a + \frac{\omega_q}{2} \sigma^z + \chi \sigma^z a^\dagger a + H_{\text{damping}}$$

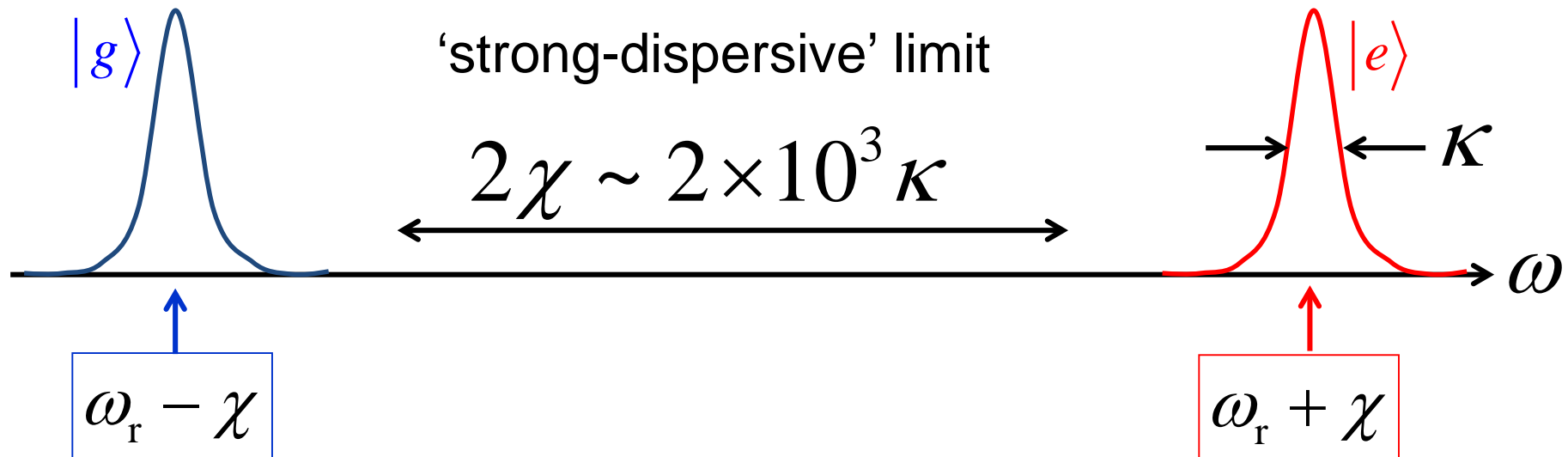
resonator

qubit

dispersive  
coupling

$$\text{cavity frequency} = \omega_r + \chi \sigma^z$$

Very easy to readout state of qubit with 99.5% fidelity in 300-400ns by measuring the cavity frequency. (QND)



# Strong-Dispersive Hamiltonian

$$H = \omega_r a^\dagger a + \frac{\omega_q}{2} \sigma^z + \chi \sigma^z a^\dagger a + H_{\text{damping}}$$

resonator

qubit

dispersive  
coupling

$$H = \omega_r a^\dagger a + \frac{\omega_q + 2\chi a^\dagger a}{2} \sigma^z + H_{\text{damping}}$$

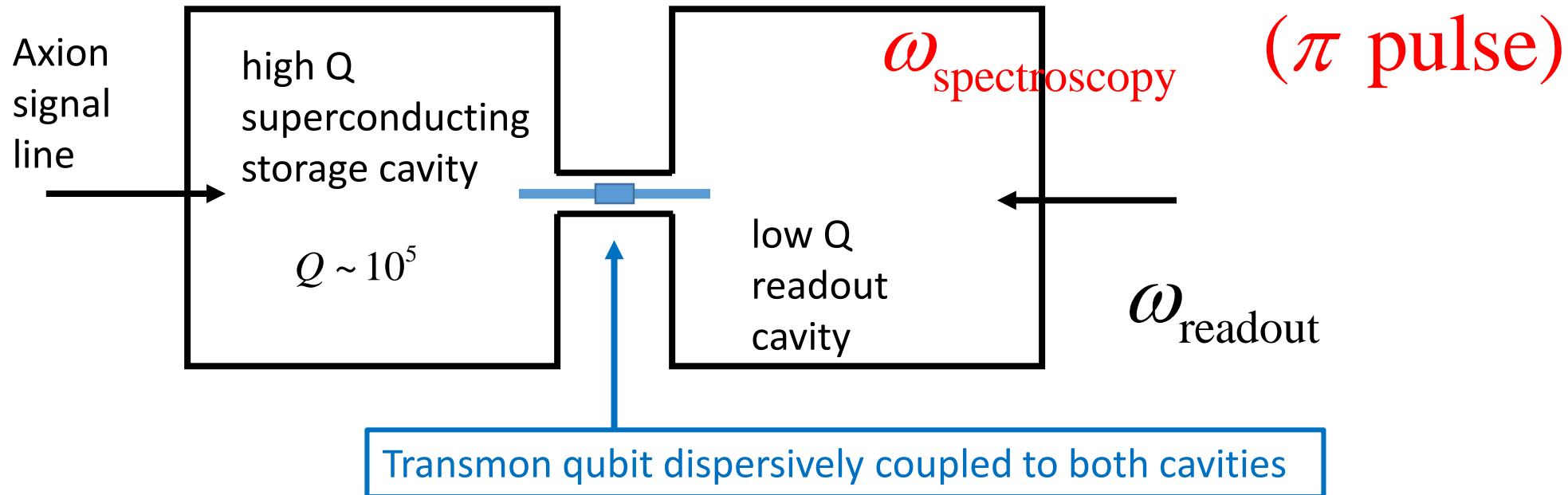
Quantized light shift of  
qubit frequency

Easy to measure the photon number in the cavity by  
measuring the qubit transition frequency.



# Usual (absorption) spectroscopy

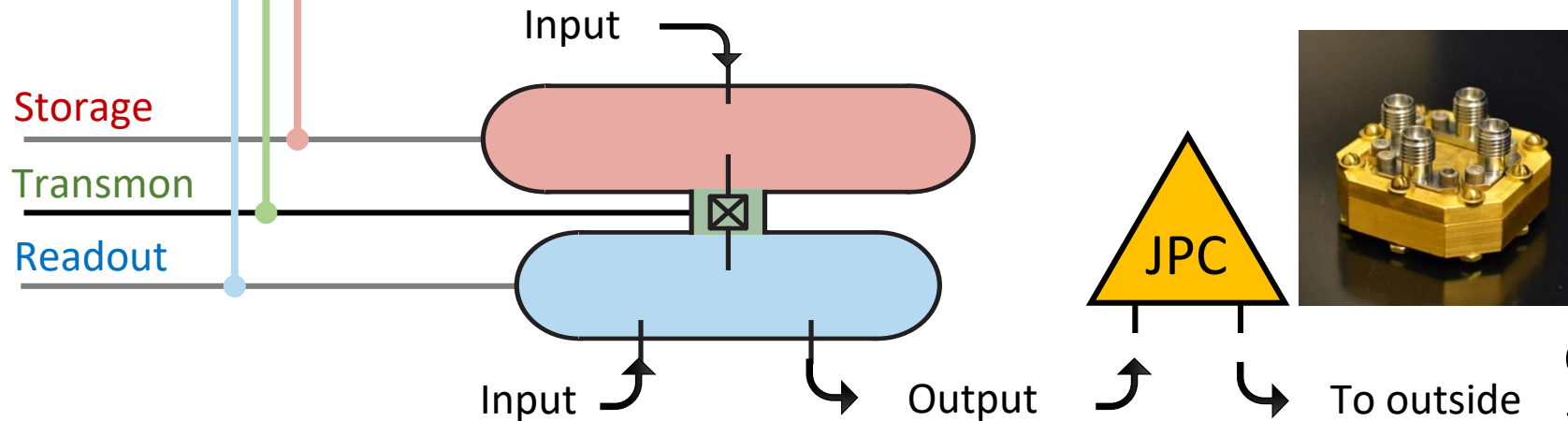
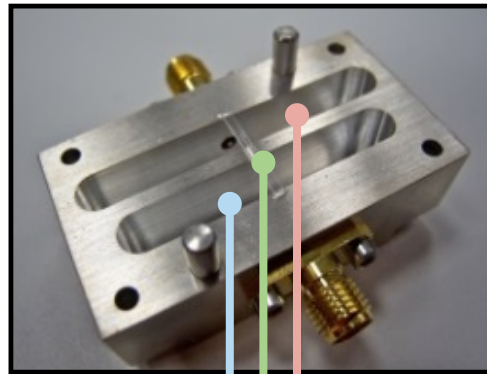
Quantum jump spectroscopy of qubit gives QND measurement of storage cavity photon number



# Quantum Jump Spectroscopy of the Qubit to detect photons in the storage cavity

8.2 GHz    6.5 GHz

$$H = \omega_s a^\dagger a + \omega_q |e\rangle\langle e| - \chi_{sa} a^\dagger a |e\rangle\langle e| - K_{ss} (a^\dagger a)^2$$

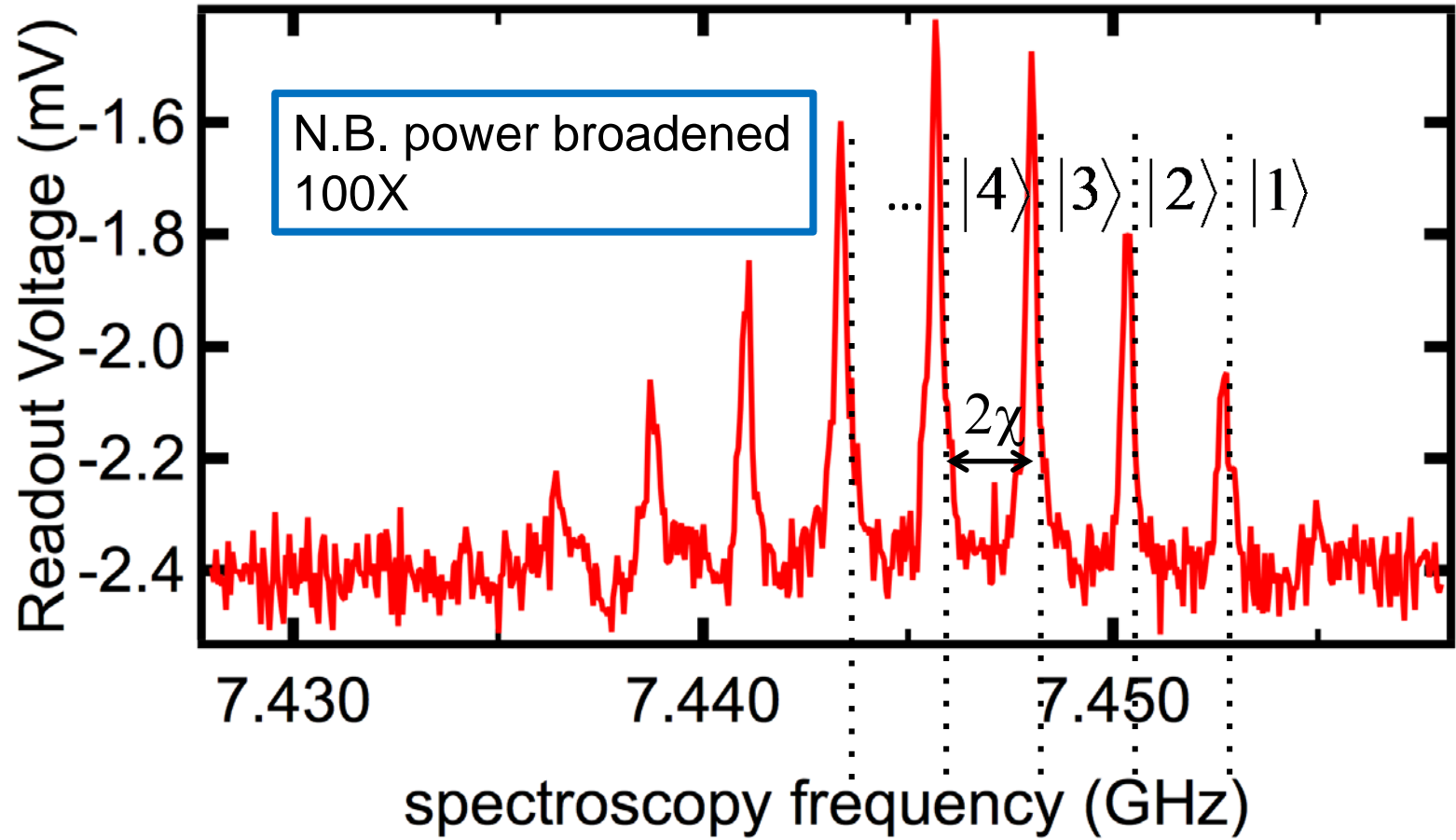


Quantum-limited amplifier  
(Devoret lab)

Ancilla (transmon) readout fidelity  $\sim 99.5\%$   
in 400 ns. QND for both qubit and storage cavity.

- quantized light shift of qubit frequency (coherent microwave state)

$$\frac{\omega_q + 2\chi a^\dagger a}{2} \sigma^z$$



New low-noise way to do axion dark matter detection using quantum jump spectroscopy of qubit?

# Thermal photon number

For 5 GHz photons in equilibrium at  $T=20\text{mK}$ ,

$$n_T \sim 3 \times 10^{-6}$$

From measurement of the quantized light shift we can bound the number of thermal photons

$$n_T < 2 \times 10^{-3}$$

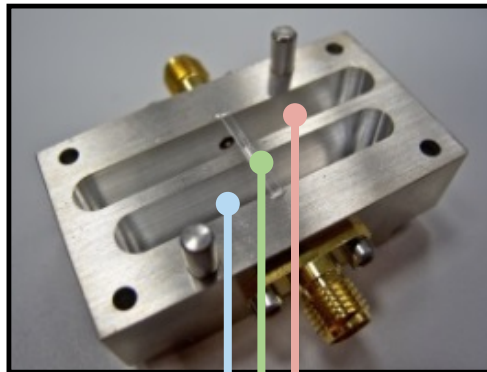
Sears et al., *Phys. Rev. B* **86**, 180504(R) (2012)

$$\bar{n}_{\text{axion}} \sim 10^{-3} - 10^{-5}$$

Qubit readout fidelity 99.5% implies  
dark count

$$\bar{n}_{\text{dark}} \sim 5 \times 10^{-3}$$

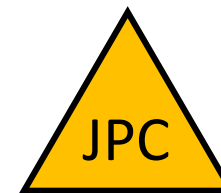
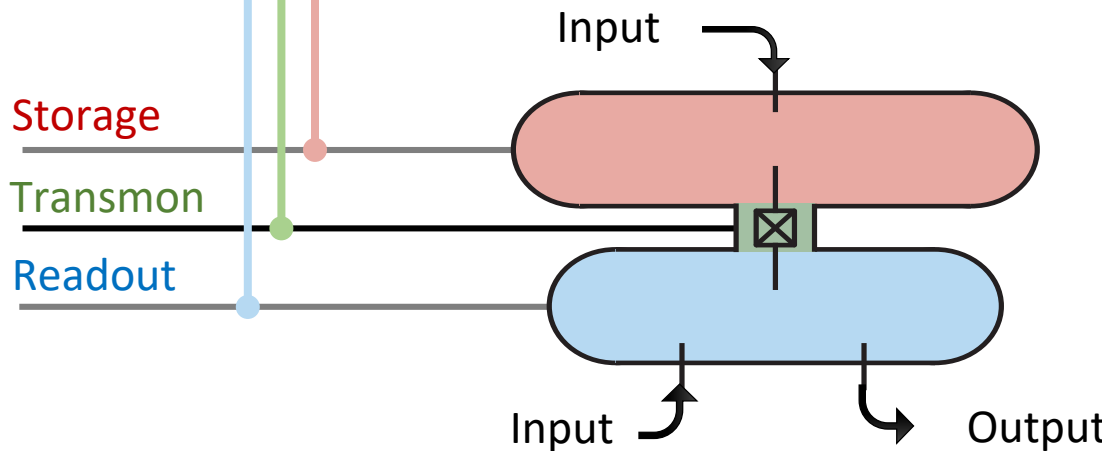
We can take advantage of the fact that the photon in the storage cavity is not absorbed during the QND measurement



$$H = \omega_r a^\dagger a + \frac{\omega_q}{2} \sigma^z + \chi \sigma^z a^\dagger a + H_{\text{damping}}$$

resonator      qubit      Dispersive coupling

(DOUBLY QND)



To outside

Quantum-limited amplifier  
(Devoret lab)

Ancilla (transmon) readout fidelity  $\sim 99.5\%$   
in 400 ns



# Thermal photon number

For 5 GHz photons in equilibrium at  $T=20\text{mK}$ ,

$$n_T \sim 3 \times 10^{-6}$$

From measurement of the quantized light shift we can bound the number of thermal photons

$$n_T < 2 \times 10^{-3}$$

Sears et al., *Phys. Rev. B* **86**, 180504(R) (2012)

$$\bar{n}_{\text{axion}} \sim 10^{-3} - 10^{-5}$$

Qubit readout fidelity  $F=99.5\%$

implies dark count

$$\bar{n}_{\text{dark}} = 1 - F \sim 5 \times 10^{-3}$$

Measurement can be repeated  $\sim 5$ - $10$  times in cavity lifetime.

## QND Readout

Veto: require  $m$  consecutive detections (not Bayes optimal)

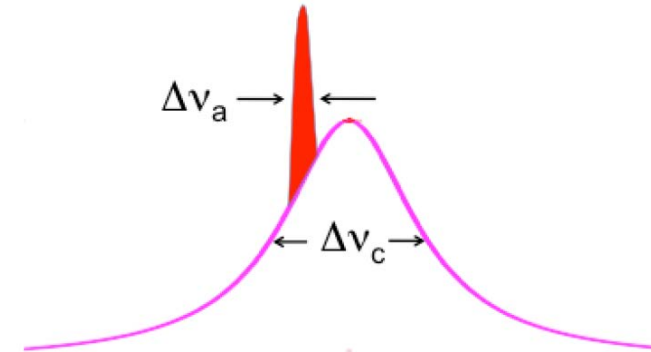
False negatives:  $F^m \sim (0.995)^m$

False positives:  $(1 - F)^m \sim (\bar{n}_{\text{dark}})^m$

$$\bar{n}_{\text{dark}}^{\text{eff}} \sim (\bar{n}_{\text{dark}})^m$$

Conclusion: Scan rate speed-up relative to coherent detection

$$R \sim \frac{\Delta\nu_{\text{axion}}}{\Delta\nu_{\text{cavity}}} \frac{\frac{1}{2} + \bar{n}_{\text{amp}} + \bar{n}_{\text{T}} + \bar{n}_{\text{axion}}}{\bar{n}_{\text{dark}}^{\text{effective}} + \bar{n}_{\text{T}} + \bar{n}_{\text{axion}}} \sim 10^2$$

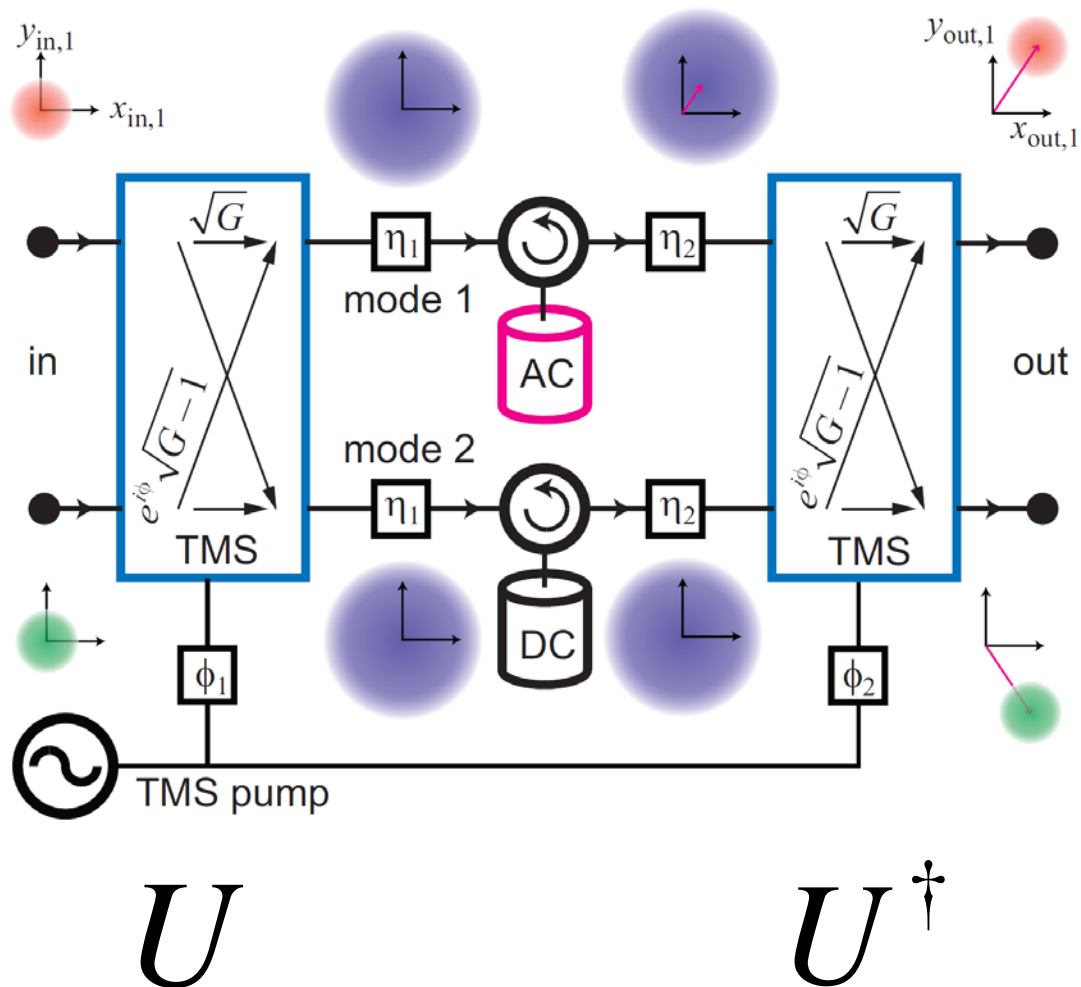


Analysis presented here is a highly simplified tutorial.

Full details of optimal acausal Bayesian filter are presented in:

Zheng et al., [arXiv:1607.02529](https://arxiv.org/abs/1607.02529) (currently being revised).

# Linear Quantum Amplification and Two-Mode Squeezing



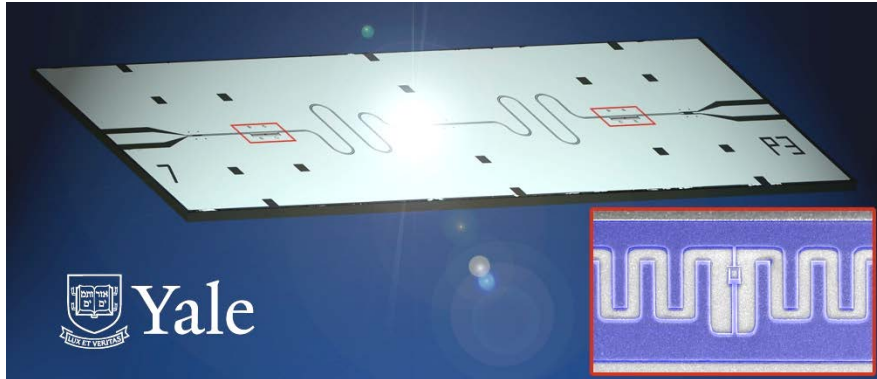
Magic of two-mode squeezing:  
vacuum noise is perfectly cancelled  
while axion signal is amplified by  $U^\dagger$ .

$S=10$  dB of squeezing has been  
achieved with current technology  
(KW Lehnert).

Risk: small transmission losses  
through the device will limit scan  
speed-up to approximately 4x unless  
they can be significantly reduced.

Circuit QED has strong-coupling microwave quantum optics has capabilities undreamt of in conventional optics.

The first electronic quantum processor (2009)



[Grover search algorithm, Deutsch Josza algorithm]

DiCarlo et al., Nature **460**, 240 (2009)



Michel  
Devoret

Rob  
Schoelkopf

Progress in quantum computation has spin-offs for table-top physics with skyscraper reach!

Many thanks to my collaborator Konrad Lehnert.

# Linear Quantum Amplification and Two-Mode Squeezing



# Quadrature Amplitudes of Quantum Signals are Incompatible Observables!

Voltage quadrature amplitudes are canonically conjugate:

$$\hat{X}(t) = \hat{X}(0) \cos(\omega t) + \hat{Y}(0) \sin(\omega t)$$

Dimensionless amplitudes:

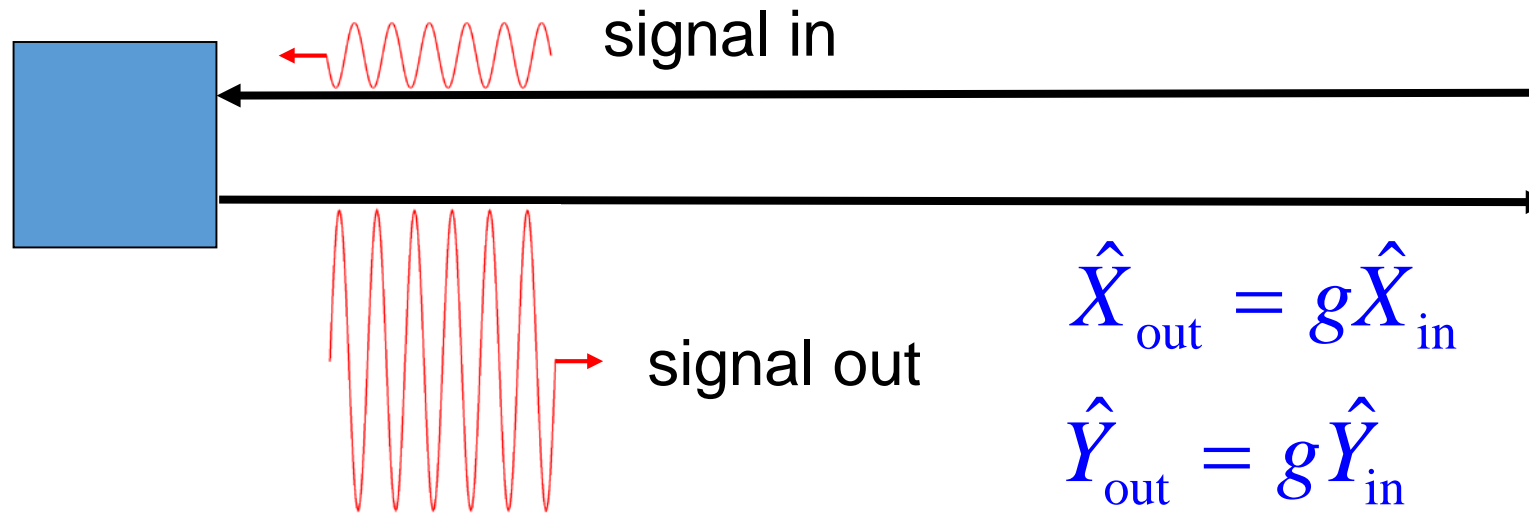
$$\hat{X} = \frac{a + a^\dagger}{2}, \quad \hat{Y} = \frac{a - a^\dagger}{2i}$$

$$[\hat{X}, \hat{Y}] = \frac{i}{2}$$

Ground (vacuum) state is  
minimum-uncertainty wave  
packet.

$$\langle 0 | X^2 + Y^2 | 0 \rangle = \frac{1}{2}$$

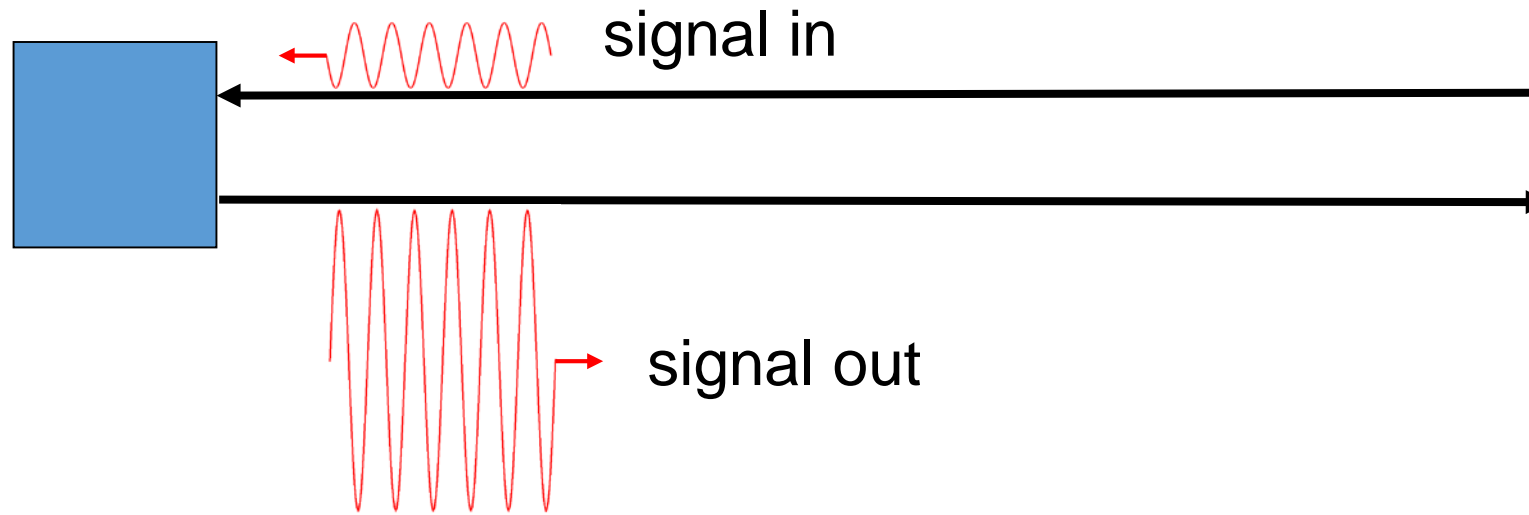
# Linear Phase-Preserving Amplification



$$\left[ \hat{X}_{out}, \hat{Y}_{out} \right] = g^2 \left[ \hat{X}_{in}, \hat{Y}_{in} \right] \neq \left[ \hat{X}_{in}, \hat{Y}_{in} \right]$$

**IMPOSSIBLE!! NOT CANONICAL**

# Linear Phase-Preserving Amplification

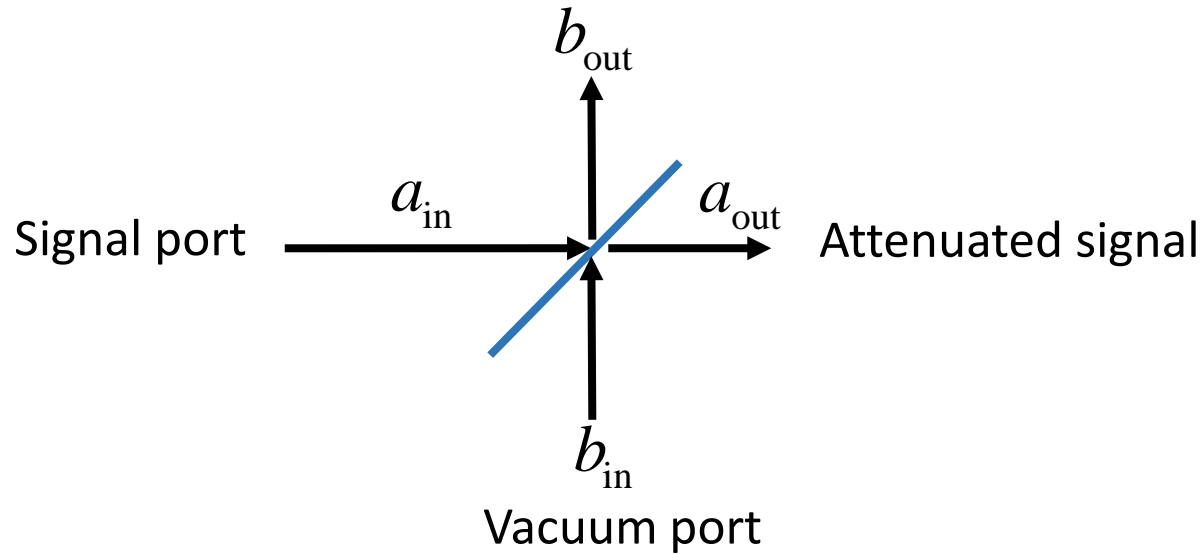


No amplifier ( $g > 1$ ) or attenuator ( $g < 1$ ) can ever do this:

$$\hat{X}_{\text{out}} = g\hat{X}_{\text{in}}$$

$$\hat{Y}_{\text{out}} = g\hat{Y}_{\text{in}}$$

# We understand beam splitters as quantum attenuators



$$\begin{pmatrix} a_{\text{out}} \\ b_{\text{out}} \end{pmatrix} = \begin{pmatrix} t & r \\ -r & t \end{pmatrix} \begin{pmatrix} a_{\text{in}} \\ b_{\text{in}} \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} a_{\text{in}} \\ b_{\text{in}} \end{pmatrix}$$

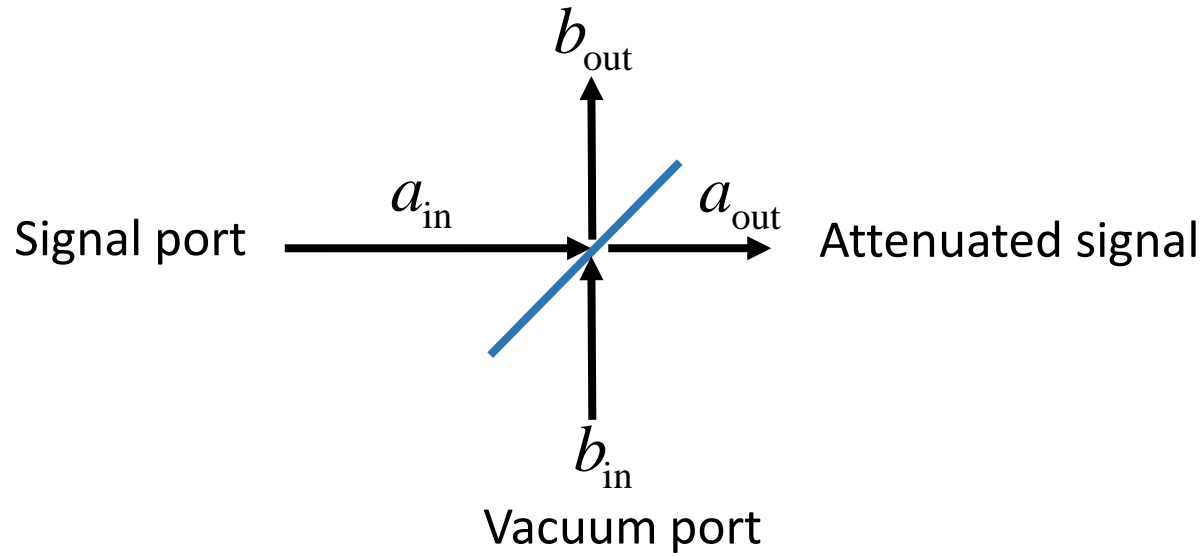
Conservation of probability = unitarity

$$t^2 + r^2 = 1$$

Unitarity requires the presence of a second (vacuum) port which adds (vacuum) noise.

$$\left[ a_{\text{out}}, a_{\text{out}}^\dagger \right] = \cos^2 \theta \left[ a_{\text{in}}, a_{\text{in}}^\dagger \right] + \sin^2 \theta \left[ b_{\text{in}}, b_{\text{in}}^\dagger \right] = 1$$

# Quantum attenuators are **SU(2)** beam splitters



$$\begin{pmatrix} a_{\text{out}} \\ b_{\text{out}} \end{pmatrix} = \begin{pmatrix} t & r \\ -r & t \end{pmatrix} \begin{pmatrix} a_{\text{in}} \\ b_{\text{in}} \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} a_{\text{in}} \\ b_{\text{in}} \end{pmatrix}$$

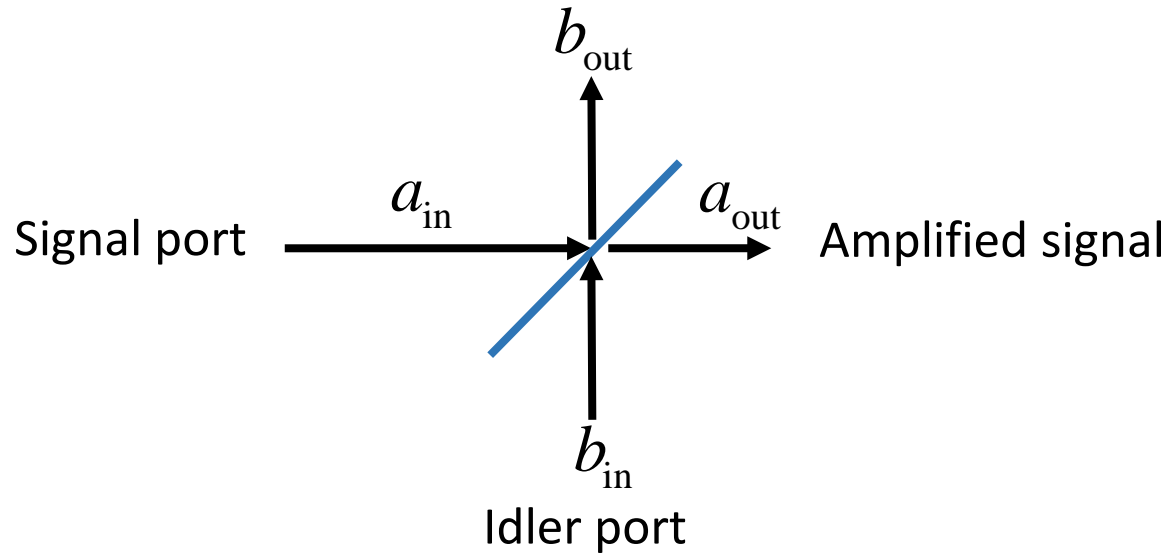
$$\left[ a_{\text{out}}, a_{\text{out}}^\dagger \right] = \cos^2 \theta \left[ a_{\text{in}}, a_{\text{in}}^\dagger \right] + \sin^2 \theta \left[ b_{\text{in}}, b_{\text{in}}^\dagger \right] = 1$$

Total vacuum noise is unaffected by the beam splitter.

$$\langle 0_{\text{in}} | X_{\text{out}}^2 + Y_{\text{out}}^2 | 0_{\text{in}} \rangle = \frac{1}{2}$$



# Quantum amplifiers are **SU(1,1)** 'beam splitters'



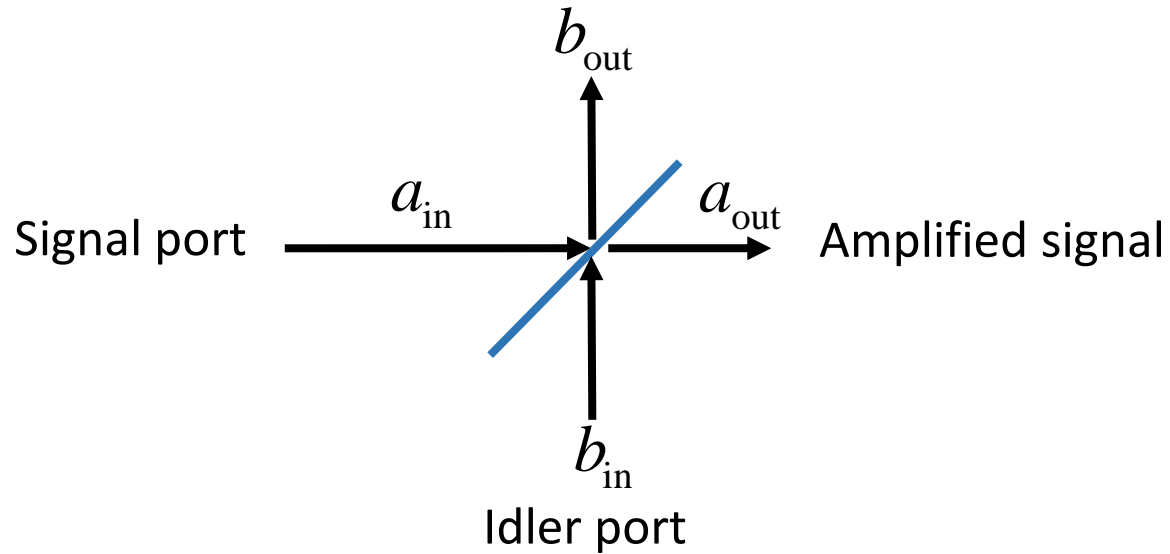
$$g^2 = G = \text{power gain}$$

$$\begin{pmatrix} a_{out} \\ b_{out}^\dagger \end{pmatrix} = \begin{pmatrix} g & \sqrt{g^2 - 1} \\ \sqrt{g^2 - 1} & g \end{pmatrix} \begin{pmatrix} a_{in} \\ b_{in}^\dagger \end{pmatrix} = \begin{pmatrix} \cosh \theta & \sinh \theta \\ -\sinh \theta & \cosh \theta \end{pmatrix} \begin{pmatrix} a_{in} \\ b_{in}^\dagger \end{pmatrix}$$

Phase conjugation

$$\left[ a_{out}, a_{out}^\dagger \right] = \cosh^2 \theta \left[ a_{in}, a_{in}^\dagger \right] + \sinh^2 \theta \left[ b_{in}^\dagger, b_{in} \right] = \cosh^2 \theta - \sinh^2 \theta = 1$$

# Quantum amplifiers are **SU(1,1)** 'beam splitters'



$$g^2 = G = \text{power gain}$$

$$\begin{pmatrix} a_{out} \\ b_{out}^\dagger \end{pmatrix} = \begin{pmatrix} g & \sqrt{g^2 - 1} \\ \sqrt{g^2 - 1} & g \end{pmatrix} \begin{pmatrix} a_{in} \\ b_{in}^\dagger \end{pmatrix} = \begin{pmatrix} \cosh \theta & \sinh \theta \\ -\sinh \theta & \cosh \theta \end{pmatrix} \begin{pmatrix} a_{in} \\ b_{in}^\dagger \end{pmatrix}$$

Phase conjugation

$$a_{out} = \sqrt{G} a_{in} + \sqrt{G-1} b_{in}^\dagger$$

Output contains amplified vacuum noise from both ports. Noise is doubled (for  $G \gg 1$ ).

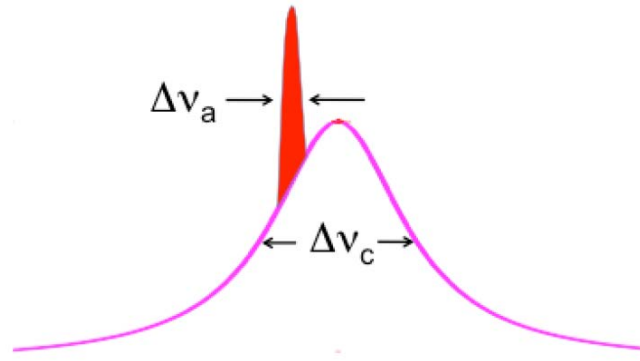
½ quantum input noise  
 ½ quantum minimum noise because of amplifying incompatible quadratures equally.

$$\frac{\langle 0_{in} | X_{out}^2 + Y_{out}^2 | 0_{in} \rangle}{G} \approx \frac{1}{2} + \frac{1}{2} = 1 \gg n_{axion}$$

# Dicke Radiometer SNR

$$\alpha_{\text{la}} = \frac{1}{2} \frac{\eta \bar{n}_{\text{axion}}}{n_{\text{amp}} + \frac{1}{2} + \bar{n}_{\text{T}}} \sqrt{(\Delta \nu_c \tau) \left( \frac{\Delta \nu_c}{\Delta \nu_{\text{axion}}} \right)}$$

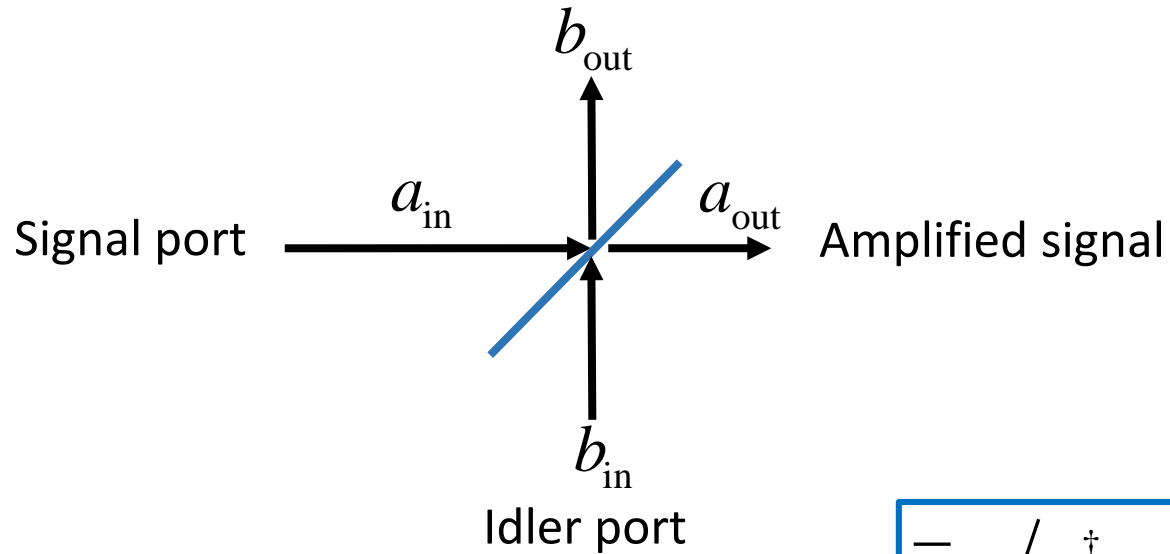
$$\approx \frac{\bar{n}_{\text{axion}} \Delta \nu_c}{2} \sqrt{\frac{\tau}{\Delta \nu_{\text{axion}}}} \quad (\text{ideal limit})$$



$$Q_a \sim 10^6$$

$$Q_c \sim 10^4 - 10^5$$

# Quantum amplifiers are **SU(1,1)** 'beam splitters'



$$g^2 = G = \text{power gain}$$

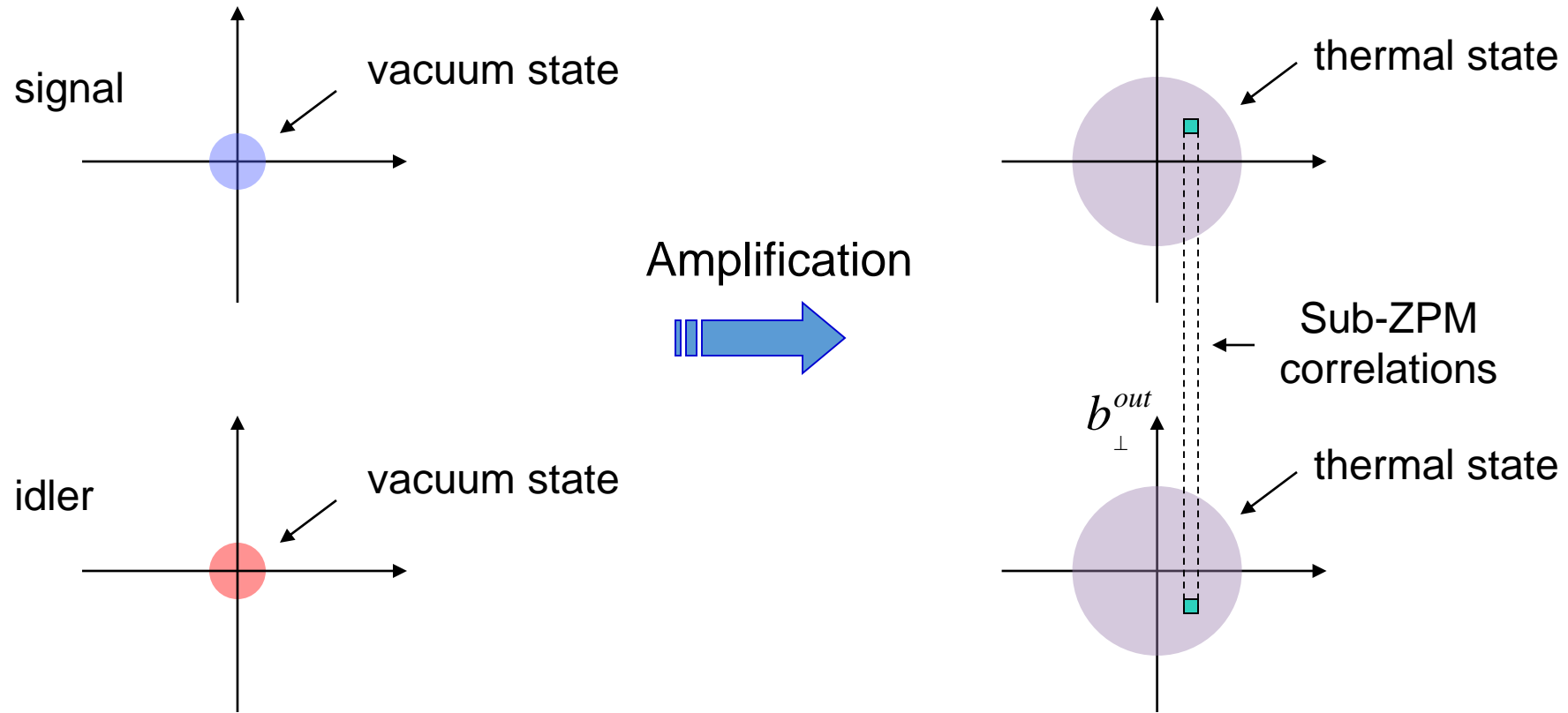
$$a_{out} = \sqrt{G} a_{in} + \sqrt{G-1} b_{in}^\dagger$$

$$\bar{n} = \langle a_{out}^\dagger a_{out} \rangle = G \langle a_{in}^\dagger a_{in} \rangle + (G-1) \langle b_{in} b_{in}^\dagger \rangle = G-1$$

$$\bar{n} = \langle b_{out}^\dagger b_{out} \rangle = G \langle b_{in}^\dagger b_{in} \rangle + (G-1) \langle a_{in} a_{in}^\dagger \rangle = G-1$$

Amplified vacuum noise yields a thermal state (Hawking radiation) in each port.

Amplified vacuum noise yields a thermal state (Hawking radiation) in each port.

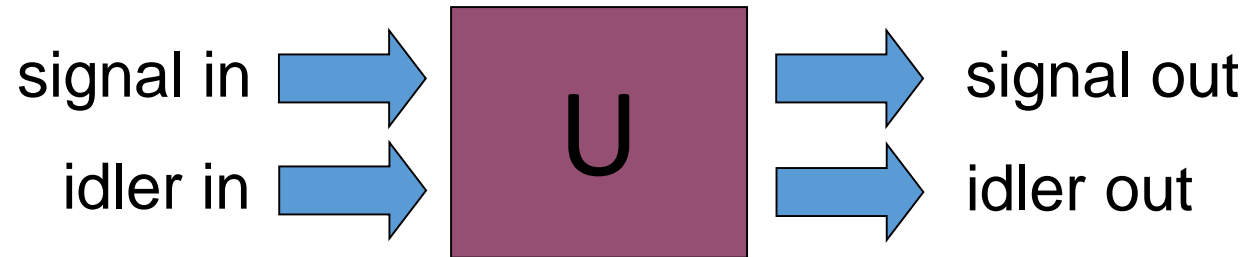


Thermal noise in each output port has large entropy.

Yet the outputs have subtle quantum correlations due to two-mode squeezing:

$$\langle a_{out} b_{out} \rangle = \sqrt{G(G-1)}$$

An ideal amplifier performs a unitary transformation from the input state to the output state.



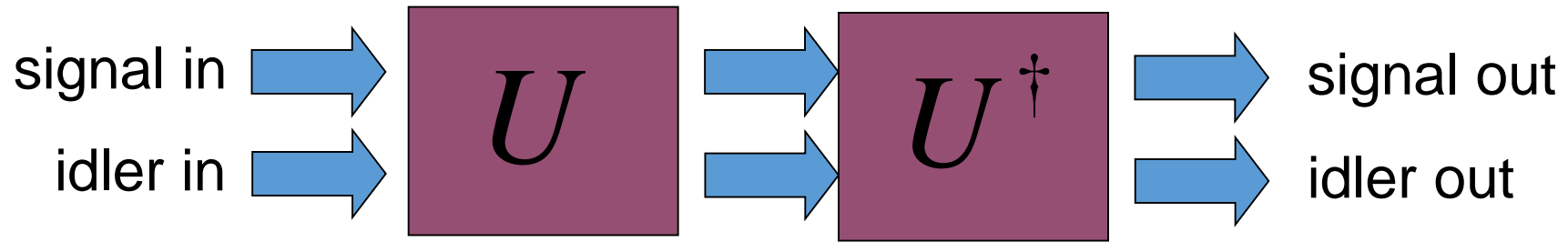
$$U \Leftrightarrow \begin{pmatrix} \cosh \theta & \sinh \theta \\ -\sinh \theta & \cosh \theta \end{pmatrix}$$

This preserves the entropy of the vacuum input state (which is zero).

Quantum entanglement of the two output beams means that each is a thermal state with positive entropy yet the entropy of the universe is still zero!

Negative entropy of quantum entanglement.

Ideal amplification is a unitary process, so can be reversed.



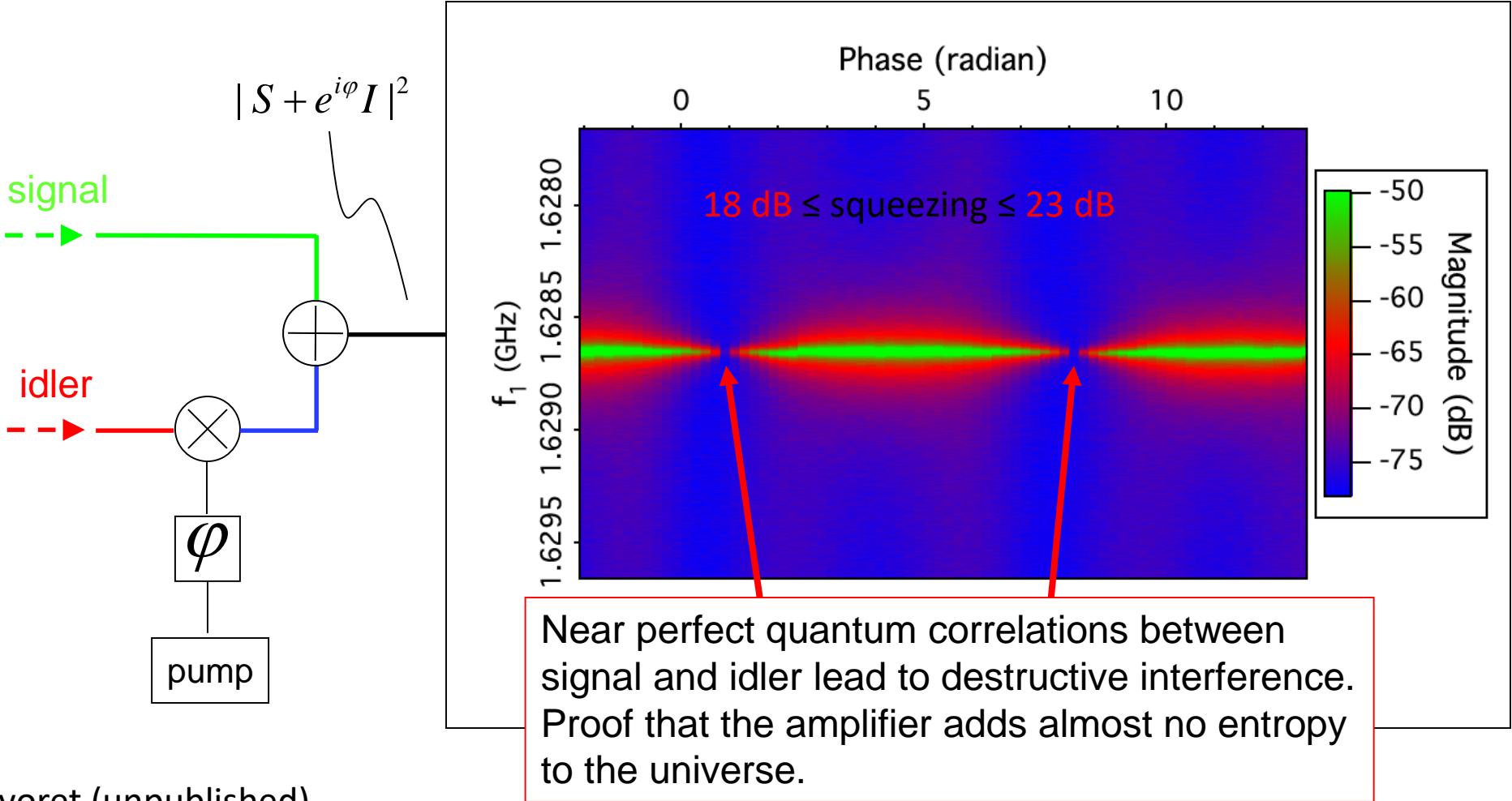
$$U(\theta) \Leftrightarrow \begin{pmatrix} \cosh \theta & \sinh \theta \\ -\sinh \theta & \cosh \theta \end{pmatrix}$$

$$U^\dagger(\theta) = U(-\theta)$$

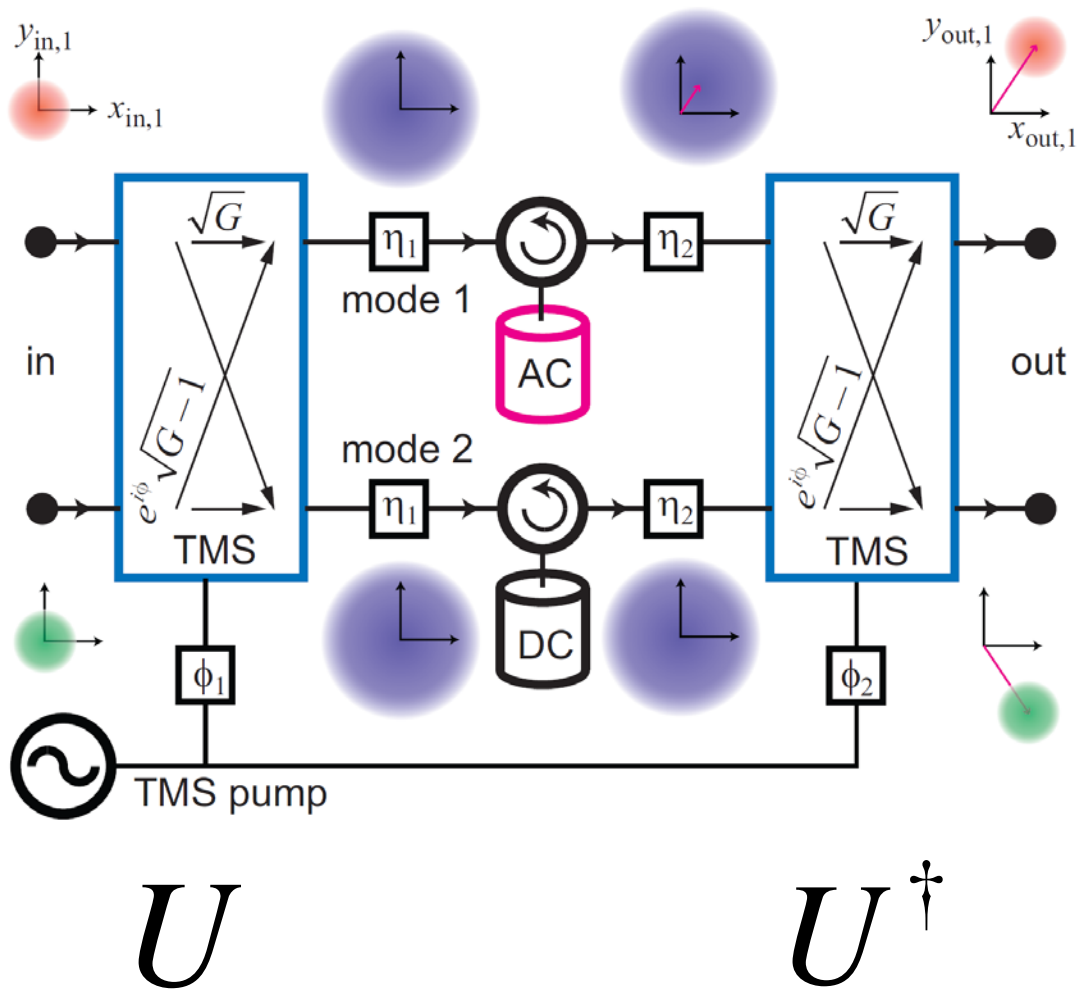
$$UU^\dagger = 1$$

vacuum in -- vacuum out!

# Interference between signal and idler: undoing the squeezing





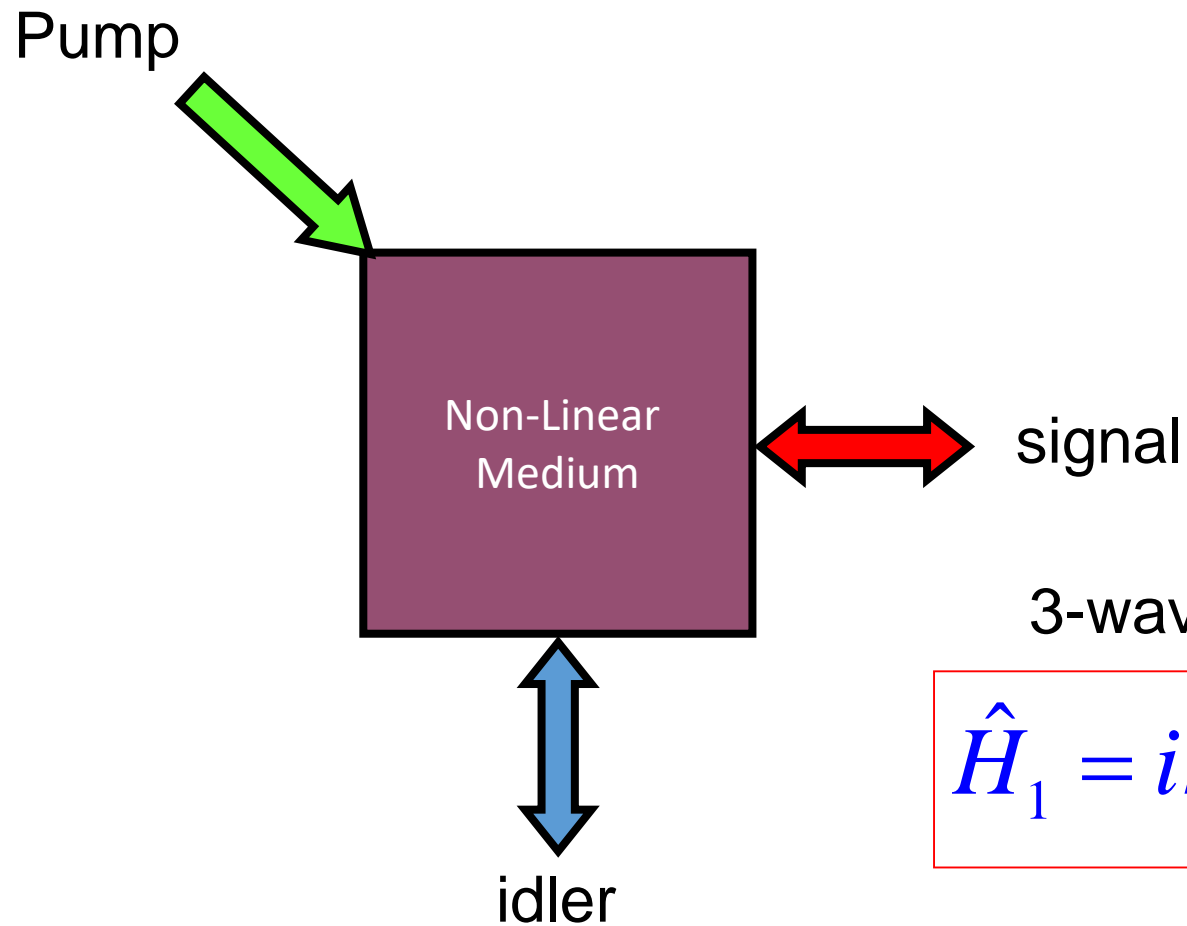


Magic of two-mode squeezing:  
vacuum noise is perfectly cancelled  
while axion signal is amplified by  $U^\dagger$ .

$S=10$  dB of squeezing has been  
achieved with current technology  
(KW Lehnert).

Risk: small transmission losses  
through the device will limit scan  
speed-up to approximately 4x unless  
they can be significantly reduced.

# Extra Slides



3-wave mixing non-linearity

$$\hat{H}_1 = i\hbar\chi (\hat{c}\hat{a}^\dagger\hat{b}^\dagger - \hat{c}^\dagger\hat{a}\hat{b})$$

$$\omega_{\text{pump}} = \omega_{\text{signal}} + \omega_{\text{idler}}$$

Energy conservation

# HAYSTAC First Results

